Influence of Viscous Dissipation on Forced Convection Heat Transfer in Thermally Developing Region of Annuli

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Abstract
Studies on forced convective heat transfer in thermally developing region of annuli including viscous dissipation and axial conduction have been reported in this thesis. The inner and outer walls have been kept at constant temperature. The governing equations are characterized by Pe, (the product of Reynolds number and Prandtl number) the Peclet number, Br, the Brinkman number and a radius ratio suitably defined for the different problems studied. Numerical solutions to the governing equations have been obtained employing Successive Accelerated Replacement (SAR) scheme for $10 \leq Pe \leq 100$, $0.1 \leq R^*$ r ≤ 10, and $-1 \leq Br \leq 1$. The inner wall Nusselt number decreases and outer wall Nusselt number increases as R * increases. For a given radius ration, both the inner and outer wall Nusselt numbers reach a limiting value, independent of Br, for all $Br \neq 0$. Of course, when $Br = 0$ the limit is the fully developed value. The ratio of fully developed inner wall Nusselt number to the outer wall Nusselt number varies linearly with the square root of outer radius to inner radius. For Br0. The statement needs a suitable modification since unbounded swing is displayed by the Nusselt number for r $Br<0$. The change is significant in the developing region and when the Peclet number is low, $Pe \geq 100$.
In the present paper heat transferred from the walls and the energy gain (or loss) by the fluid up to a desired axial distance has been defined which serve the purpose of the average Nusselt number more conveniently. A key feature is that when axial conduction or viscous dissipation or both are included, the heat transferred from (to) the walls differs from the energy gained (or lost) by the fluid.

1 INTRODUCTION

Flow and heat transfer through ducts of different cross-sections has been under investigation for the past few decades. The classical Graetz problem {see, Kays and Crawford [1]} deal with flow through pipes. Other geometries include annuli, channel, and ducts of square, rectangular or triangular cross-section. Recent applications in compact heat exchanger and proton exchange membrane fuel cells motivated the studies that involved annuli and channels. An excellent review of the studies prior to 1970 is available in Shah and London [2] and in Kakac, Shah and Aung [3]. The commonly employed thermal boundary conditions are 1. Specified constant heat flux. 2. Specified constant wall temperature. 3. Boundary conditions of four kinds as elaborated in Shah and London. [2] Relatively more attention is paid on pipe and channel geometries. Simplest of the studies involve assuming fully developed flow and thermal fields, neglecting viscous dissipation of incompressible Newtonian fluid Barletta [4, 5, and 6] has considered viscous dissipation in channel configurations. Recently Mitrovic, [11 and 16], Avdin and Avci [7, 8, 9, and 10] studied thermally developing flow including viscous dissipation but neglected axial conduction.

1.1 Classification Of Internal Flow Forced Convection Heat Transfer Studies

on steady internal flow laminar forced convective heat transfer within the framework of continuum, incompressible, single phase flow of a Newtonian fluid may be broadly classified based on the geometry, type of fluid, thermal boundary conditions etc.

1.1.1 Assumptions Regarding Flow and ThermalFields,

Developed or Developing: The investigations reported assumed the following conditions to exist. 1) Hydrodynamically and thermally fully developed flow. 2) Hydrodynamically fully developed and thermally developing flow. 3) Hydrodynamically and thermally 2 developing flow. Mostly, the developing fields have been dealt with, invoking boundary layer approximations.

1.1.2 Additional Effects:

Additional effects include, 1) viscous dissipation, 2) applied magnetic or electric fields 3) internal heat sources 4) Marangoni convection, and 5) buoyancy etc. As has been stated earlier, present day applications such as electronic cooling and fuel cells involve heat transfer in annulus, subjected to constant temperature thermal boundary conditions and viscous dissipation may play an important role. Studies on these aspects are further motivated by noting from the available literature, viscous dissipation effects poses certain issues that need to be resolved /reconciled. The fully developed Nusselt number value for annulus subjected to uniform and equal wall temperatures, neglecting viscous dissipation is 7.54 (see, Shah and London [2]). Viscous dissipation, with reference to internal flows through, pipes, annuli, channel etc., the walls of which have been kept at constant temperature, is characterized by the Brinkman number defined as, ( ) 2 m i w u Br k T T μ = − (1.1) Where, Ti the inlet temperature, Tw the constant wall temperature, um the average fluid velocity, k the thermal conductivity of wall, μ the dynamic viscosity of fluid, According to Eq. (1.1), Br>0 represents a fluid being cooled and Br< 0. Thus, continuity in heat transfer with axial distance, and Br is an issue. Also, wall heat transfer vis-à-vis. fluid heat gain or loss need to be examined.

2 LITERATURE REVIEW ON LAMINAR FORCED CONVECCTION WITH VISCOUSDISSIPATION IN DUCT

The problem of forced convection in an annulus is a classical problem that has been revisited in recent years in connection with the cooling of electronic equipments using materials involving hyper porous media or micro channels. The review presented here describes representative developments describing laminar forced convection in ducts including viscous dissipation. The main geometries under the category of ducts include pipes, channels and annuli.

2.1 Effect of Axial Conduction When axial conduction term in the energy equation is included, the thermal field depends on a non-dimensional parameter, referred to as Peclet number, Pe, which is a product of Reynolds number, Re and Prandtl number, Pr, even after introducing the normalized axial distance, X * . Pe = Re.Pr X * = X/ (Pe) (1.2) The non-dimensional X = x/ Dh. x is the dimensional axial distance and Dh a length scale (say, hydraulic diameter) used to non-dimensionalize. Re and Pr are defined as, Re = (ρ um Dh)/μ and Pr = (μ cp)/k (1.3) In Eq. (1.3), um is some reference velocity and ρ, the density, μ, the dynamic viscosity, cp, the specific heat and k, the thermal conductivity of the fluid.

2.2 Viscous Dissipation Viscous dissipation is characterized by the Brinkman number Br, defined in Eq.(1.1).Effects of viscous dissipation on heat transfer for flow between parallel plates have been studied by Hwang, Knieper and Fan [22], for uniform heat flux boundary condition. The developing temperature profiles and the local Nusselt numbers have been presented for Br, –1, –0.5, 0, 0.5 and 1. Barletta [19] analyzed the combined free and forced convection flow in a parallel plate vertical channel in the fully developed region by taking into account the effect of viscous dissipation. Barletta used the perturbation series method, the perturbation parameter being proportional to viscous dissipation and buoyancy. Nusselt number results presented are valid when both the walls are at equal temperature and when they are at different temperatures. Results of Barletta are valid only in the conduction limit (i.e., all convective terms have been dropped), though include buoyancy and viscous dissipation. Further, there is no distinction when the wall temperatures differ by varying degrees. The limiting Nusselt number of 17.5 when viscous dissipation is included is applicable when the wall temperatures are set equal, in Barletta's result. Aydin and Avci [8] studied the laminar heat convection in Poiseuille flow of a Newtonian fluid with constant properties by taking viscous dissipation into account for constant heat flux and constant wall temperature boundary conditions. They investigated a) both hydrodynamically and thermally fully developed flow and b) hydrodynamically fully developed but thermally developing flow neglecting axial conduction. Results of Aydin and Avci are distinctly different for the cases of fluid being heated and being cooled. Further, the Nusselt number displays an unbounded swing when the fluid is being heated even though, Orhan and Avci assumed a uniform temperature at the entry. This is in contrast to the difference attributed by Barletta and Magyari [18] in the axial evolution of Nusselt number when the entrance is preceded by an adiabatic preparation (length) and when the entrance temperature is assumed to be uniform. When dissipation is included, it is to be noted that the fully developed value of Barletta [19] for equal wall temperatures is independent of Br whereas the result of Cheng and Wu [20] depends on Br when the wall temperatures are unequal. Dependence of Nusselt number on Br in the thermally developing region for equal wall temperatures as well as constant wall heat flux can be found in very recent studies by Aydin and Avci[8]. From the results of Aydin and Avci, as can be expected in general, Nusselt number does depend on Br for the case of constant heat flux in the developing as well as in the developed region. Results of Aydin and Avci for Nusselt number in the developing region for constant wall temperature also depend on Br. However, the results of Aydin and Avci do not extend up to the fully developed region and hence it is difficult to state whether the fully developed Nusselt number reaches 17.5, independent of Br, as in Barletta [19]. In the present investigation related to viscous dissipation, the issues of a) dependence of Nusselt number on Brinkman number in the developing region, b) Nusselt number reaching a limit independent of Br in the developed region when the channel walls are subjected to constant, equal temperatures and c) the behavior of the Nusselt number with Br when the walls are subjected to unequal temperatures have been addressed. It is noted that the Nusselt number displays an unbounded swing when the walls are at unequal temperatures. Similarly, the unbounded swing in the Nusselt number occurs due to viscous dissipation when the fluid is being heated, even when the wall temperatures are equal. Thus, it is of interest to examine the behavior when the walls are kept at unequal temperatures and viscous dissipation is included .

3 LACUNAE IN THE PAST STUDIES ON LAMINAR FORCED CONVECTION IN DUCTS

From the literature review given in § 1.2, it is evident practically no studies involved flow through annuli including viscous dissipation. Laminar forced convection in thermally developing region. Also, no studies including viscous dissipation considered axial conduction except the recent studies by Ramjee [47]. Ramjee obtained numerical solutions employing two-dimensional steady governing equations including viscous dissipation to describe simultaneously developing flow and thermal fields in channels. Ramjee obtained numerical solutions employing the Successive Accelerated Replacement Scheme (SAR) [12, 13, 14, and 15]. 1.4 NUMERICAL SCHEMES Numerical studies on laminar forced convection in the entrance region of a channel when both velocity and temperature fields are developing employing twodimensionalNavier-Stokes and thermal energy equations including viscous dissipation are computationally intensive. The ellipticity of the equations, makes applying the downstream boundary condition an iterative process. Boundary layer approximation makes the method amenable for a marching forward procedure. Habchi and Acharya [23] used the implicit finite-difference scheme, to solve the energy conservation equation with boundary layer approximation. Naito and Nagano [24] solved the full Navier-Stokes equations and energy equation numerically by using Successive Over-Relaxation (SOR) [25] method. Nguyen [26, 27] used Alternating Direction Implicit (ADI) [28, 29] and Quadratic Upwind Interpolation for Convective Kinematics (QUICK) [30] methods to solveNavier-Stokes and energy equations in the finite difference form. The SemiImplicit Method for Pressure Linked Equations-Revised (SIMPLER) [31] algorithm with a staggered grid system is employed by Jeng, Chen and Aung [32]. Crank-Nicholson semi-implicit scheme is used by Krishnan and Sastri [33] to solve the energy equation. Min et al. [34] solved the discritized momentum and energy equations by using a line-byline Tri-Diagonal Matrix Algorithm (TDMA) [35], with the pressure equation solved by using a line SOR. The successive accelerated replacement (SAR) scheme also has been employed successfully for a wide class of problems. SAR scheme is essentially the non-linear over relaxation method due to Lew [36], Lieberstein [37] and Dellinger [38]. Lew [36] and Dellinger [38] applied the SAR scheme for solving non-linear ordinary differential equations. Dellinger's scheme differs from the non-linear over relaxation method essentially in choosing the relaxation factor. Satyamurty [39] demonstrated the applicability of the SAR scheme for solving system of partial differential equations in the study of two-dimensional natural convection heat transfer in porous media and this scheme has been extensively applied by Satyamurty and Marpu [40], Marpu and Satyamurty [41], Satyamurty and Marpu [42], Marpu [43], Sharma [44], Prakash Chandra [45] and Satyamurty and Prakash Chandra [46] for natural convection heat transfer in porous media including variable fluid properties and non-Darcy flow descriptions. Sharma applied the SAR scheme successfully for three-dimensional natural convection problems in porous media. SAR scheme has been chosen to obtain numerical solutions to the problems studied in the present thesis.

4 SCOPE AND OBJECTIVES

The objective of present investigation is to study laminar forced convection heat transfer in the entrance region of annuli subjected to constant wall temperature including viscous dissipation. The problem has been studied under the following progressively increasing complexity.

1. Nusselt number in the conduction limit.
2. Fully developed flow, but thermally developing field neglecting axial conduction but including dissipation.
3. Fully developed flow including dissipation and axial conduction in the energy equation.

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